Indian Statistical Institute, Bangalore

B. Math (Hons.) Third Year

Second Semester - Differential Geometry II

Final Exam Date: May 10, 2018

Maximum Marks: 50 Duration: 3 hours

1. Define the following notions:- [3 + 3 + 3 + 3 + 3 = 15]
   (a) Involutive distribution
   (b) Integral submanifold
   (c) Complete vector field
   (d) Two vector fields ϕ-related
   (e) Left-invariant vector field.
2. Prove that S^3 is diffeomorphic to $SU(2)$. [7]
3. Define a tensor field of type (r,s) on a smooth manifold M. Let w be a 1-form on M and define $F : \chi(M) \times \chi(M) \to C^\infty(M)$ by $F(X,Y) = X(w(Y))$. Is F a $(2,0)$- tensor field? [2 + 4 = 6]
4. Give an example of a manifold which is not orientable and justify your answer. [2 + 3 = 5]
5. The stereographic projection $\varphi : (\mathbb{S}^m \setminus \{(1,0,0,...,0)\}, < \ , \ >_{\mathbb{R}^{n+1}}) \to (\mathbb{R}^m, \frac{4}{(1+|x|^2)^2} < \ , \ >_{\mathbb{R}^m})$ is given by $\varphi(x_0, x_1, x_2...x_m) = \frac{1}{1-x_0}(x_1, x_2, ...x_m)$. Show that φ is an isometry. [7]
6. Let $SO(m)$ be the special orthogonal group equipped with the metric $< X, Y >= \frac{1}{2}\text{trace } (X^t \cdot Y)$. Prove that $< \ , \ >$ is left-invariant and that for vector fields $X, Y \in T_eSO(m)$, we have $\nabla_X Y = \frac{1}{2}[X,Y]$. [5 + 5 = 10]